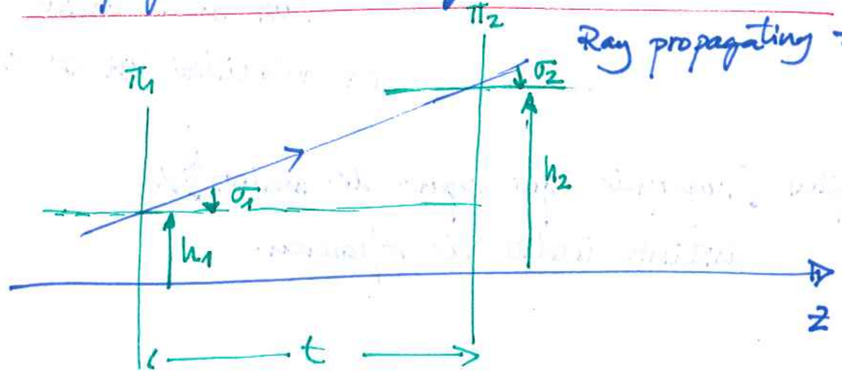
12/09/2016

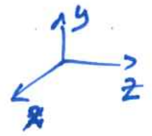
NETWORK: FULL PARALLAX IMAGING

1.- GEOMETRICAL OPTICS: RAY TRACING.

1.1.- Propagación de un rayo en un medio.

Ray propagating through a medium.

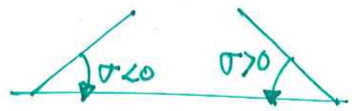




. $\sigma \equiv$ lo tomamos desde el rayo hasta el eje.

$\sigma > 0$ el rayo converge hacia el eje

$\sigma < 0$ el rayo diverge del eje.



Si consideramos la aproximación paraxial $\sigma \ll$: (small angles)

$$\text{sen}\,\sigma \simeq \text{tg}\,\sigma \simeq \sigma$$

Entonces:

$$\sigma_2 = \sigma_1$$

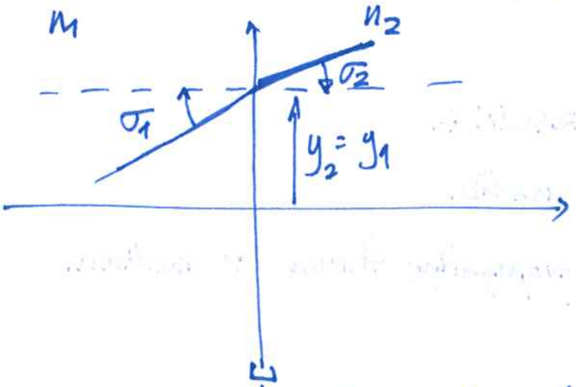
$$\sigma_2 = \frac{y_2 - y_1}{-t} \longrightarrow y_2 = y_1 - \sigma_1 t$$

$$\left.\begin{aligned} y_2 &= A y_1 + B\sigma_1 \\ \sigma_2 &= C y_1 + D\sigma_1 \end{aligned}\right\}$$

$$\begin{pmatrix} y_2 \\ \sigma_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & -t \\ 0 & 1 \end{pmatrix}}_{} \begin{pmatrix} y_1 \\ \sigma_1 \end{pmatrix}$$

$\mathbb{T}$ = matriz de transmisión.

1.2.- Propagación de un rayo en dos medios diferentes: REFRACCIÓN.



$$\boxed{n_i = \frac{c}{v_i}} \equiv \text{Refractive index.}$$

$v_i \equiv$ velocidad de la luz en el medio

$c \equiv$ velocidad luz en vacío

↳ dioptrio (superficie que separa dos medios de distinto índice de refracción.

∘ REFRACTION LAW:

$$\boxed{n_1 \cdot \text{sen}\, \sigma_1 = n_2 \cdot \text{sen}\, \sigma_2},$$

so . . .

$$\left\{ \begin{aligned} y_2 &= y_1 \\ \sigma_2 &= \frac{n_1}{n_2}\sigma_1 \end{aligned} \right. \qquad \begin{pmatrix} y_2 \\ \sigma_1 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}}_{} \begin{pmatrix} y_1 \\ \sigma_2 \end{pmatrix}$$

$R \equiv$ matriz de refracción.

— RESUMEN —

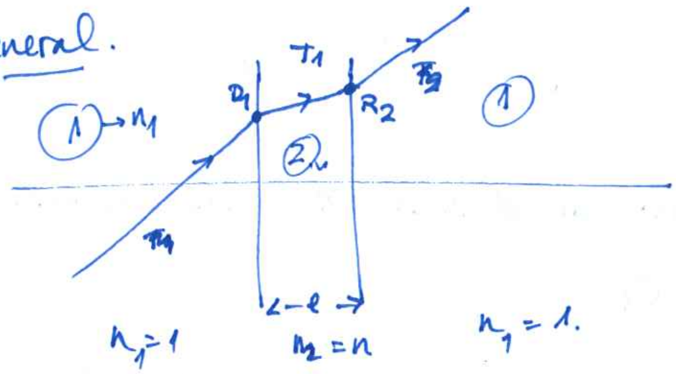
$$T \equiv \begin{pmatrix} 1 & -t \\ 0 & 1 \end{pmatrix} \equiv \text{Transmission matrix}$$

$$\Rightarrow \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$P \equiv \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix} \equiv \text{Refraction matrix.}$$

1.3.- Propagación en varios medios: lámina planoparalela.

- General.



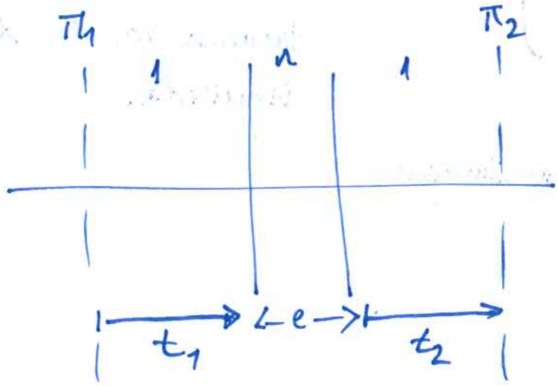
- $$M_{12} = \begin{pmatrix} 1 & 0 \\ 0 & n/1 \end{pmatrix} \begin{pmatrix} 1 & -e \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/n \end{pmatrix} = \begin{pmatrix} 1 & -\frac{e}{n} \\ 0 & 1 \end{pmatrix}$$

$$\quad\quad\quad\quad\; R_2 \quad\quad\quad\quad T_1 \quad\quad\quad\quad R_1$$

↳ matriz de propagación entre dos medios.

$$\begin{pmatrix} y_2 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{e}{n} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ \sigma_1 \end{pmatrix} \Rightarrow \left.\begin{aligned} y_2 &= y_1 - \frac{e}{n}\sigma_1 \\ \sigma_2 &= \sigma_1 \end{aligned}\right\}$$

- Entre dos planos concretos



$$M = \begin{pmatrix} 1 & -t_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -\frac{e}{n} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -t_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -\left(t_1 + t_2 + \frac{e}{n}\right) \\ 0 & 1 \end{pmatrix}$$

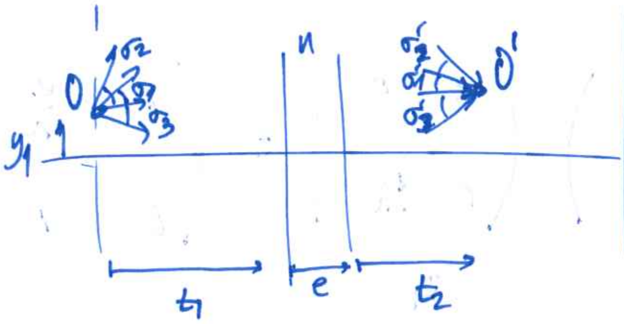
$$y_2 = y_1 - \left(t_1 + t_2 + \frac{e}{n}\right)\sigma_1$$

$$\sigma_2 = \sigma_1$$

$\rightsquigarrow$ NOTA: en principio vemos que $y_2 \neq y_1$ y que $\sigma_2 = \sigma_1$.

$y_2 = f(\sigma_1)$

¿ Podemos encontrar un punto en ~~el~~ donde ~~t~~ y sea la misma y cambie σ ? $\rightarrow$ $y_2 \neq f(\sigma_1)$



si $y_2 \neq f(\sigma_1) \Rightarrow$

$\Rightarrow t_1 + t_2 + \frac{e}{n} = 0$

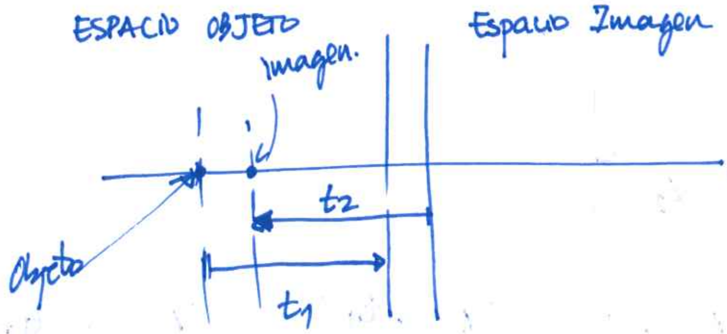
$\rightarrow$ esta es la condición de formación de imagen.

Bajo la condición de formación de imágenes, el elemento B de la matriz Traslación – Refracción – Traslación es 0.

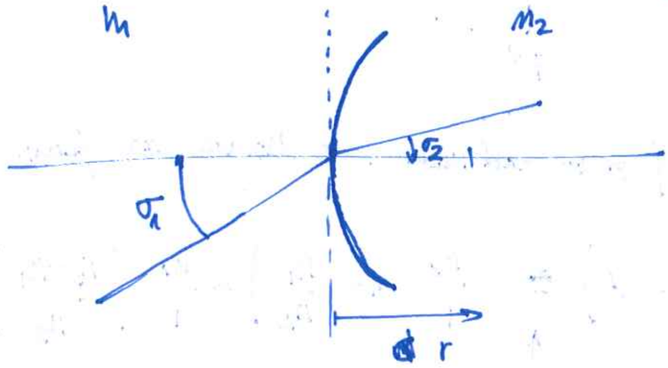
$\rightarrow B=0 \rightarrow$ Relación de conjugación O – O'

Así

$$\left.\begin{aligned} t_1 + t_2 + \frac{e}{n} &= 0 \\ t_2 &= -t_1 - \frac{e}{n} \end{aligned}\right\} \rightarrow t_2 < 0 \rightarrow \text{en ~~el~~ espacio imagen va de derecha a izquierda.}$$



1.4.- Refracción en un dioptrio esférico.



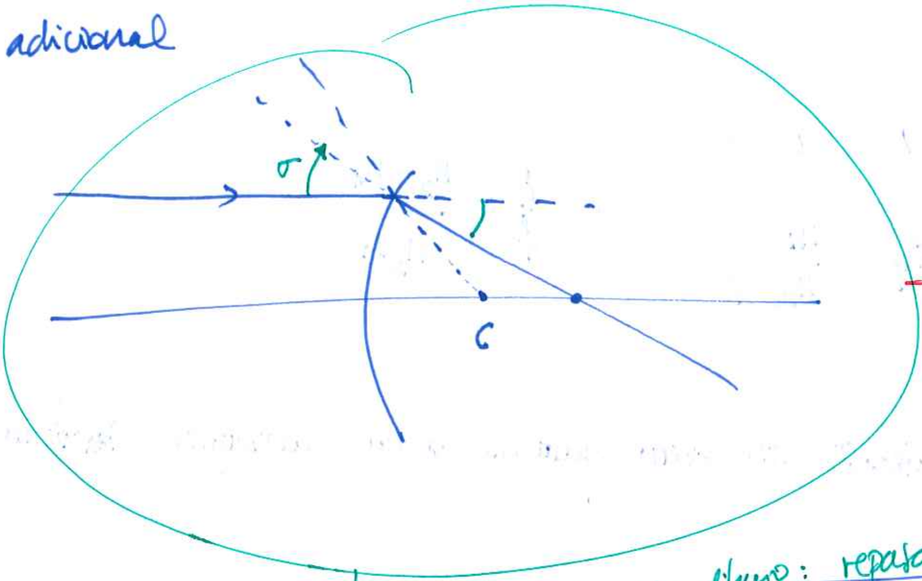
$$\left.\begin{aligned} y_2 &= A y_1 + B \sigma_1 \\ \sigma_2 &= C y_1 + D \sigma_1 \end{aligned}\right\}$$

· Condiciones para la formación de imágenes:

Refracción: $B = 0$, $y_2 = y_1$.

$$S = \begin{pmatrix} 1 & 0 \\ C & \frac{n_1}{n_2} \end{pmatrix}$$, ¿cuál es el valor de C?

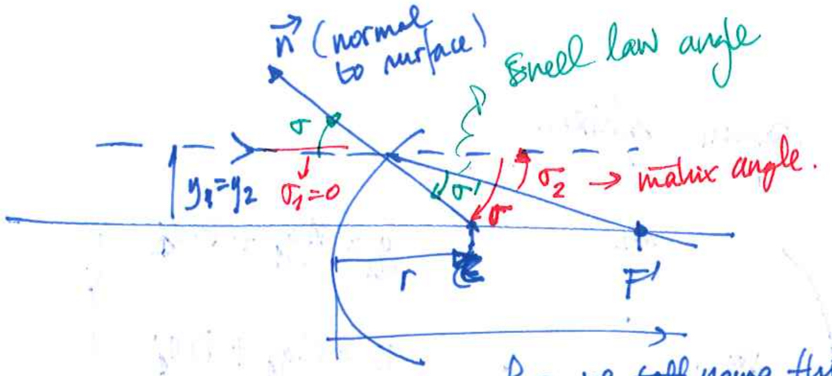
Para conocer el valor de C debemos ~~empla~~ emplear un rayo adicional



→ Ver dibujo detrás.

↳ ~~Vaya mierda de dibujo: repasar y mejorar.~~

→ ~~Ver rayo de refracción y rayos a la sup.~~



f = we ~~call~~ name this distance as focus.

$$\sigma_2 = (\sigma' - \sigma) = \sigma\left(\frac{n_1}{n_2} - 1\right) = \frac{y_1}{r}\left(1 - \frac{n_1}{n_2}\right) = \frac{y_1}{r}\left(\frac{n_2 - n_1}{n_2}\right)$$

$$n_1\sigma = n_2\sigma' \qquad \sigma = -\frac{y_1}{r}$$

$$S = \begin{pmatrix} 1 & 0 \\ C & \frac{n_1}{n_2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{n_2 - n_1}{n_2 \cdot r} & \frac{n_1}{n_2} \end{pmatrix}$$

$$\sigma_2 = \frac{y_1}{f} = \frac{y_1}{r}\left(\frac{n_2 - n_1}{n_2}\right) \longrightarrow \frac{1}{f} = \frac{1}{r}\left(\frac{n_2 - n_1}{n_2}\right),$$ por lo que

$$S = \begin{pmatrix} 1 & 0 \\ 1/f & \frac{n_1}{n_2} \end{pmatrix}, \quad \frac{1}{f} = \frac{n_2 - n_1}{n_2 \cdot r}$$

EXERCISE 1. Demonstrate as every parallel beam converges [illegible] in the focus.

$$\begin{pmatrix} h_2 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1/f & \frac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} h_1 \\ \sigma_1 \end{pmatrix} \Rightarrow \begin{cases} h_2 = h_1 \\ \sigma_2 = \frac{1}{f} h_1 + \frac{n_1}{n_2}\sigma_1 \end{cases}$$

$$M = \begin{pmatrix} 1 & -s' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/f & 1/n \end{pmatrix} = \begin{pmatrix} 1-\frac{s'}{f} & -\frac{s'}{n} \\ 1/f & 1/n \end{pmatrix}$$

$$\begin{pmatrix} y_2 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} 1-\frac{s'}{f} & -\frac{s'}{n} \\ 1/f & 1/n \end{pmatrix} \begin{pmatrix} y_1 \\ \sigma_1 \end{pmatrix} \Longrightarrow \begin{aligned} y_1\left(1-\frac{s'}{f}\right) - \sigma_1 \frac{s'}{n} &= y_2 \\ \frac{1}{f} y_1 + \frac{1}{n}\sigma_1 &= \sigma_2 \end{aligned}$$

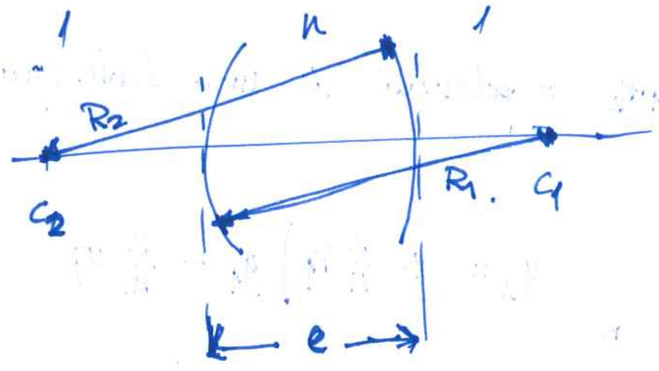
Demostración: condiciones del problema.

C1) $\sigma_1 = 0 \;\forall\; y_1 \longrightarrow y_1\left(1-\frac{s'}{f}\right) = y_2$

C2) $y_2 = 0$ (dónde corta al eje?) $\longrightarrow y_1\left(1-\frac{s'}{f}\right) = 0$

$\left(1-\frac{s'}{f}\right) = 0 \to s' = f$ c.q.d.

1.5.- Thick Lens



$$TL = \begin{pmatrix} 1 & 0 \\ 1/f_2 & n \end{pmatrix} \begin{pmatrix} 1 & -e \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/f_1 & \frac{1}{n} \end{pmatrix} =$$

$$\frac{1}{f_1} = \frac{n-1}{n r_1} = \frac{P_1}{n}$$

$f_1 \equiv$ focal [m]

$P_1 =$ power [m^{-1}] (dioptries).

$$\frac{1}{f_2} = \frac{1-n}{r_2} = P_2 \quad , \quad r_2 < 0.$$

This way, the matrix for thick lenses becomes as:

$$TL = \begin{pmatrix} 1 - \frac{e}{n} P_1 & -e/n \\ \underbrace{P_2\left(1 - \frac{e}{n} P_1\right) + P_1}_{P} & 1 - \frac{e}{n} P_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Nota: $B \neq 0$, porque esta es la matriz de la lente. La condición $B = 0$ es entre planos conjugados; es decir, entre el plano objeto y su correspondiente plano imagen.

$C = \frac{1}{f} \equiv$ equivale a la ~~focal~~ potencia del sistema de lente gruesa.

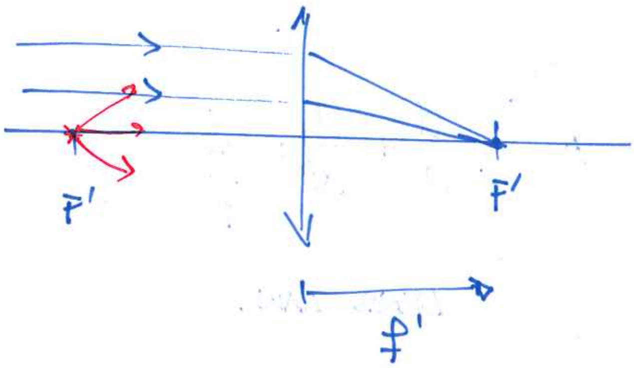
$P = P_1 + P_2 - \frac{e}{n} P_1 \cdot P_2$ = potencia de una lente gruesa.

$$\begin{pmatrix} y_2 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} TL \end{pmatrix} \begin{pmatrix} y_1 \\ \sigma_1 \end{pmatrix} \longrightarrow \begin{aligned} y_2 &= \left(1 - \frac{e}{n} P_1\right) y_1 - \frac{e}{n} \sigma_1 \\ \sigma_2 &= P y_1 + \left(1 - \frac{e}{n} P_2\right) \sigma_2 \end{aligned}$$

1.5.1- Thin lens aproximation: when $e \longrightarrow 0$

$$T_nL = \begin{pmatrix} 1 & 0 \\ P & 1 \end{pmatrix} \qquad \begin{aligned} y_2 &= y_1 \\ \sigma_2 &= P \cdot y_1 + \sigma_1 \\ P &= P_1 + P_2 \end{aligned}$$

(thin lens ...)



$$M=\begin{pmatrix} 1 & -f \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}=\begin{pmatrix} 0 & -f \\ -1/f & 1 \end{pmatrix}$$

$y_2 = -f\sigma_1 \quad ; \quad \sigma_1 = 0$

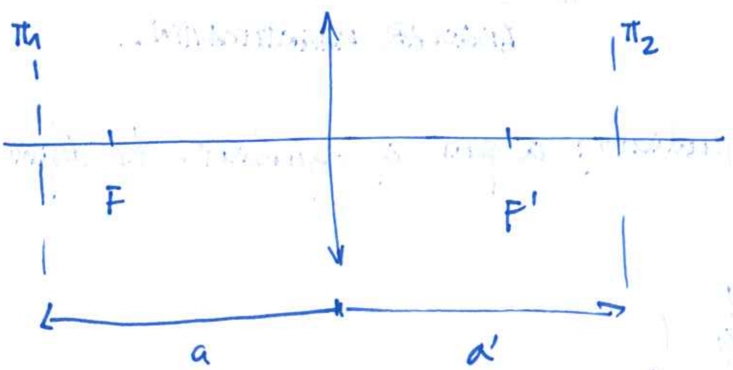
$\sigma_2 = -\frac{1}{f} y_1 + \sigma_1$

What happens if the rays start in F instead of ~~convergia~~ being parallel?

$$M=\begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}\begin{pmatrix} 1 & -f \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} 1 & -f \\ -1/f & 0 \end{pmatrix}$$

$$\left.\begin{aligned} y_2 &= y_1 - f\sigma_1 \\ \sigma_2 &= -\frac{1}{f} y_1 \end{aligned}\right\} \quad C.I \to y_1 = 0 \longrightarrow \boxed{y_2 = -f\sigma_1}$$

$\hookrightarrow \boxed{\sigma_2 = 0}$ los rayos salen paralelos.



$$M_{12}=\begin{pmatrix} 1 & -a' \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}\begin{pmatrix} 1 & +a \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} 1-\frac{a'}{f} & +a-a'-\frac{aa'}{f} \\ 1/f & 1+\frac{1}{f} \end{pmatrix}$$

The condition for imaging ≡ conjugated planes: $B = 0$ $\begin{pmatrix} A & B=0 \\ C & D \end{pmatrix}$,

so...

$$+a - a - \frac{aa'}{f} = 0 \longrightarrow \boxed{-\frac{1}{a} + \frac{1}{a'} = \frac{1}{f}}$$

LENS LAW.

then, if two planes are conjugated OBJ → IMG, the matrix:

$$M_{12} = \begin{pmatrix} 1-\frac{a'}{f} & 0 \\ 1/f & 1+\frac{a}{f} \end{pmatrix} \rightsquigarrow \begin{pmatrix} y_2 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} 1-\frac{a'}{f} & 0 \\ 1/f & 1+\frac{a}{f} \end{pmatrix} \begin{pmatrix} y_1 \\ \sigma_1 \end{pmatrix},$$

so:

$$y_2 = \left(1 - \frac{a'}{f}\right) y_1 \longrightarrow \boxed{M = \frac{y_2}{y_1} = \frac{a'}{a} = A}$$

LATERAL MAGNIFICATION

$$\boxed{D = \gamma = \frac{\sigma_2}{\sigma_1} = \frac{a}{a'}} \quad \gamma = \frac{1}{M}$$

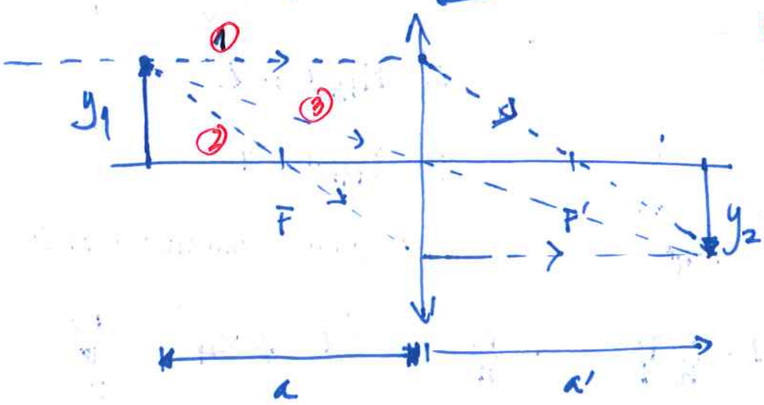
ANGULAR MAGNIFICATION..

- Correspondence equations: a pair of equations to define the system.

$$\left.\begin{array}{ll} 1) & -\frac{1}{a} + \frac{1}{a'} = \frac{1}{f'} \\ 2) & M = \frac{y'}{y} = \frac{a'}{a} \end{array}\right\}$$

1.6.- Graphic Ray Tracing

1.6.1.- Real image $a > f$

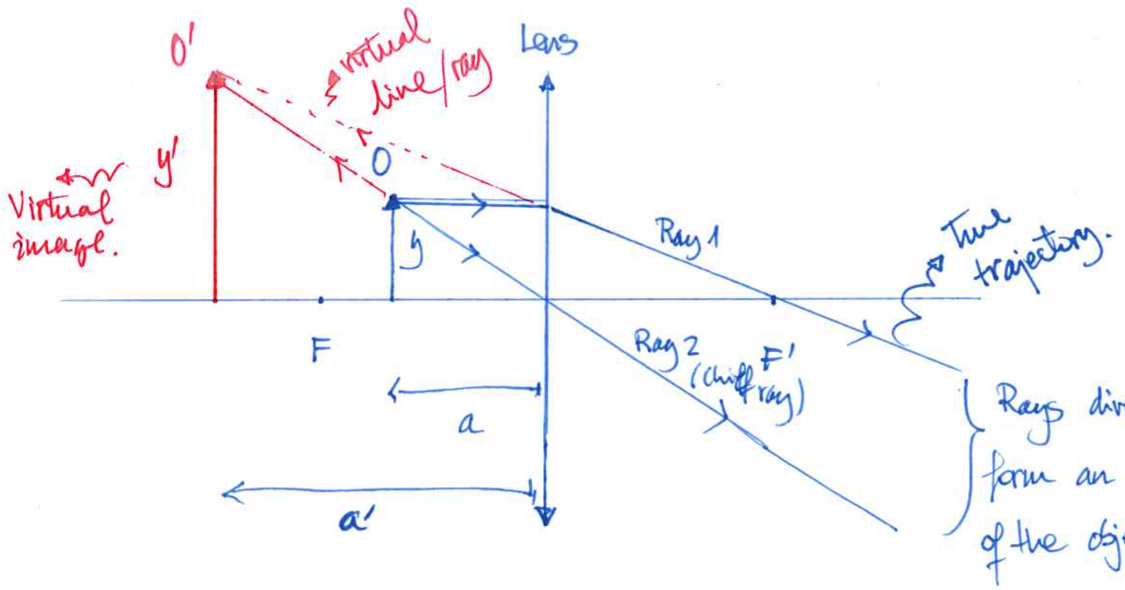


1) Los rayos que inciden paralelos al eje se refractan hacia el foco imagen.

2) ~~El~~ rayo que pasa por el foco objeto se refracta paralelo al eje.

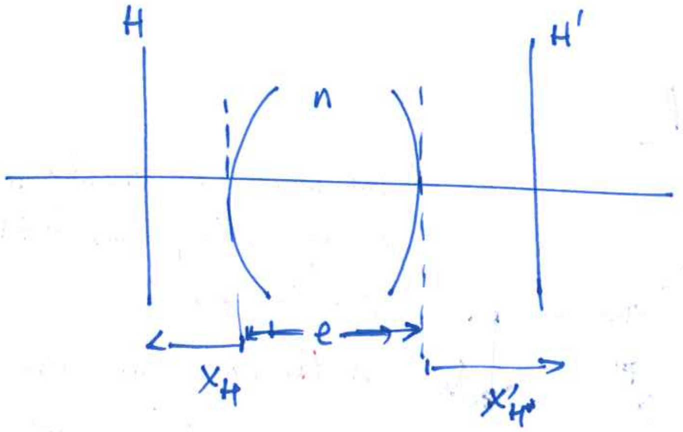
3) El rayo que pasa por el centro no se desvía de su trayectoria.

1.6.2.- Virtual image $a < f$



- The image point O' it is not a real point since the true rays (Ray 1 and Ray 2) do not converge along the actual trajectories. O' can be only "viewed" with an additional lens in such a way that ~~the~~ for this additional lens the diverging rays seem to proceed from a plane placed at a distance a' from the main lens.

1.7.- Thick Lens Revisited.



We ask...:

1) Two conjugated planes
2) $M_{HH'} = 1$.
3) $\gamma_{HH'} = 1$

↳ We name theseones : PRINCIPAL PLANES

$$M_{12} = \begin{pmatrix} 1 & -x'_H \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1-\frac{e}{n}P_1 & -\frac{e}{n} \\ P & 1-\frac{e}{n}P_2 \end{pmatrix} \begin{pmatrix} 1 & +x_H \\ 0 & 1 \end{pmatrix} \overset{①}{=}$$

$$\overset{①}{=} \begin{pmatrix} 1-\frac{e}{n}P_1 - x'_H P & x_H\left(1-\frac{e}{n}P_1\right) - \frac{e}{n} - x'_H\left(Px_H + 1 - \frac{e}{n}P_2\right) \\ P & Px_H + 1 - \frac{e}{n}P_2 \end{pmatrix}$$

so... we want to :

2) $A = 1 \implies x'_H = -\frac{e}{n}\frac{P_1}{P}$

3) $D = 1 \implies x_H = \frac{e}{n}\frac{P_2}{P_1}$

① → from 2) and 3) we demonstrate $B = 0$.

and finally:

$$\rightarrow M_{HH'} = \begin{pmatrix} 1 & 0 \\ P & 1 \end{pmatrix}$$ = MATRIX FOR PRINCIPAL PLANES

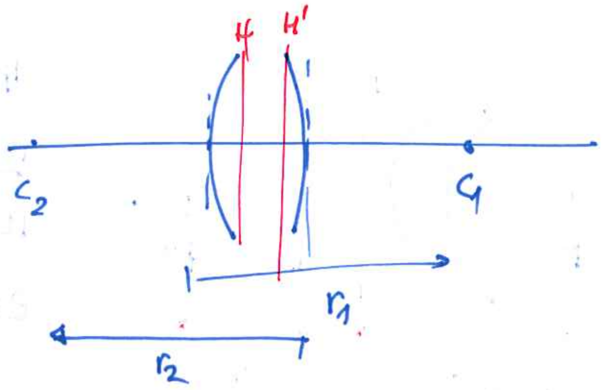
Example 1 BICONVEX LENS

$r_1 = 100$ mm

$r_2 = -100$ mm

$e = 6$ mm

$n = 1'5$



- $P_1 = \dfrac{n-1}{r_1} = 5'0\ D$; • $P = P_1 + P_2 - \dfrac{e}{n} P_1 \cdot P_2 = 9'99\ D$

- $P_2 = \dfrac{1-n}{r_2} = 5'0\ D$

NOTE: The focal length is the distance $\overline{H'F'}$.

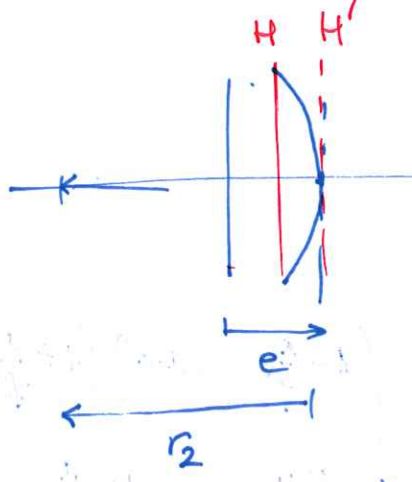
~~$P = P_1 + P_2 + \ldots D$~~

Then...

$X_H = +2$ mm

$X_{H'} = -2$ mm

Example 2: PCX

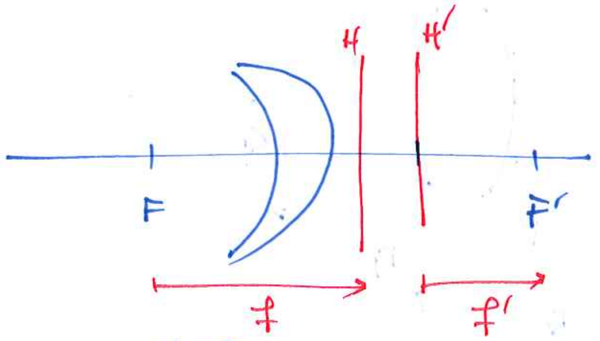


$r_1 = \infty$ (plane)

$\left.\begin{array}{l} P_1 = 0 \\ P_2 = 5'00\ D \\ P = P_2 = 5'00\ D \end{array}\right\} \rightarrow \begin{array}{l} X_H = 4\text{ mm} \\ X_{H'} = 0 \end{array}$

EXAMPLE 3: OPHTHALMIC LENS

We have a meniscus:



$r_1 = -1000$ mm

$r_2 = -50$ mm.

$n = 1'5$

$e = 4$ mm.

$P_1 = -5'00\,D$

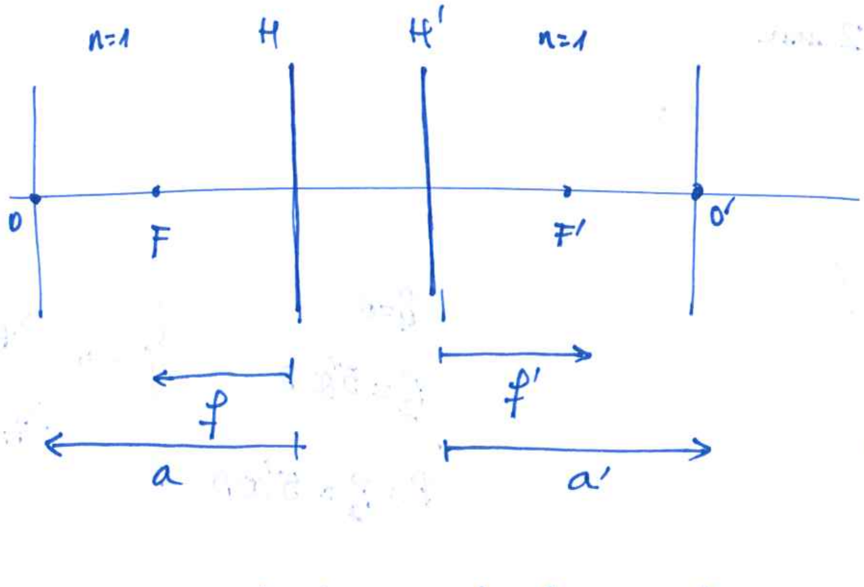
$P_2 = +10'00\,D$

$P = 5'13\,D$

$x_H = +5'2$ mm

$x_{H'} = +2'6$ mm

18.- CORRESPONDENCE EQUATIONS:



$$M_{OO'} = \begin{pmatrix} 1 & -a' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{a'}{f'} & (a-a')\left(1 + \frac{a}{f'}\right) \\ 1/f' & 1 + \frac{a}{f'} \end{pmatrix}$$

→ Correspondence matrix.

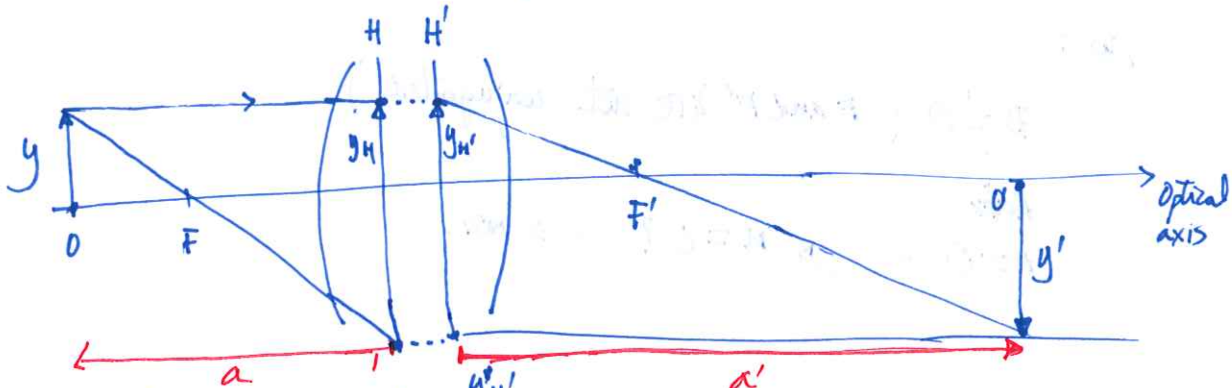
• Image condition: $B=0 \longrightarrow (a-a')\left(1-\frac{a}{f}\right)=0$, so

$$\boxed{-\frac{1}{a}+\frac{1}{a'}=\frac{1}{f}} \Rightarrow$$ conjugation equation.
(Gauss - lens equation).

• Magnification (lateral): $A = 1-\frac{a'}{f'} = \text{Magnification} = \frac{a'}{a}$

• Angular magnification: $D = \gamma = \frac{1}{M} = \frac{a}{a'}$
if $n=n'$

1.8.1.- Ray tracing: We can simplify an optical system (complex) by means of their principal planes



Principal planes: $M=1=\frac{y'_{H'}}{y_H}$